First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

MAT223 - Differential Equations I
(Persamaan Pembezaan I)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN (7) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH (7) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **FIVE (5)** questions.

*[**Arahan** : Jawab **LIMA (5)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

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Question 1

(a) State the order of the given ordinary differential equations where $y = y(x)$. Determine whether the equation is linear or nonlinear:

(i) $x^5 y^{(4)} - x^3 y'' + 6y = 0.$

(ii) $y'' = \sqrt{1 + (y')^2}.$

(iii) $(\sin x)y''' - (\cos x)y' = 2.$

(b) Suppose a person carrying a flu virus returns to an isolated village of 1000 people. The rate at which the disease spreads is proportional to the number of interactions between the number of people who have the flu and the number of people who have not yet been exposed to it. Determine a differential equation for the number of people $x(t)$ who have contracted the flu. **Do not solve** the differential equation.

(c) Solve the differential equation

$$\frac{dy}{dx} = \frac{xy^2 - \cos(x)\sin(x)}{y(1 - x^2)}, \quad y(0) = 2,$$

where $y = y(x)$.

[100 marks]

Soalan 1

(a) Nyatakan peringkat bagi persamaan pembezaan yang diberi di mana $y = y(x)$. Tentusahkan sama ada persamaan tersebut linear atau tak linear:

(i) $x^5 y^{(4)} - x^3 y'' + 6y = 0.$

(ii) $y'' = \sqrt{1 + (y')^2}.$

(iii) $(sin\, x)y''' - (cos\, x)y' = 2.$

(b) Andaikan seorang pembawa virus selsema pulang ke sebuah kampung terpencil yang mempunyai 1000 orang penduduk. Kadar penyakit tersebar adalah berkadaran dengan bilangan interaksi antara bilangan penduduk yang menghidap selsema dan bilangan penduduk yang masih tidak terdedah kepada penyakit tersebut. Tentusahkan persamaan pembezaan bagi bilangan penduduk $x(t)$ yang dijangkiti selsema. ***Jangan selesaikan*** *persamaan pembezaan tersebut.*

(c) *Selesaikan persamaan pembezaan*

$$\frac{dy}{dx} = \frac{xy^2 - cos(x)\, sin(x)}{y(1-x^2)}, \quad y(0) = 2,$$

di mana $y = y(x)$.

[100 markah]

Question 2

(a) Find the solution of the initial value problem

$$4y'' + 4y' + 17y = 0, \quad y(0) = -1, \quad y'(0) = 2.$$

(b) Find a particular solution of the non-homogeneous differential equation

$$4y'' + 36y = \csc(3x).$$

(Hint: $\csc(3x) = \frac{1}{\sin(3x)}$).

(c) Given the standard form of a linear n-th order differential equation as

$$y^{(n)} + \frac{a_{n-1}}{a_n} y^{(n-1)} + \cdots + \frac{a_2}{a_n} y^{(2)} + \frac{a_1}{a_n} y^{(1)} + \frac{a_0}{a_n} y = f(x).$$

Write the complementary solution, particular solution and the Wronskian.

[100 marks]

Soalan 2

(a) *Cari penyelesaian bagi masalah nilai awal*

$$4y'' + 4y' + 17y = 0, \quad y(0) = -1, \quad y'(0) = 2.$$

(b) *Cari suatu penyelesaian khusus bagi persamaan pembezaan tak homogeny*

$$4y'' + 36y = \csc(3x).$$

(Petunjuk: $\csc(3x) = \frac{1}{\sin(3x)}$).

(c) *Diberi bentuk am kepada persamaan pembezaan linear peringkat-n sebagai*

$$y^{(n)} + \frac{a_{n-1}}{a_n} y^{(n-1)} + \cdots + \frac{a_2}{a_n} y^{(2)} + \frac{a_1}{a_n} y^{(1)} + \frac{a_0}{a_n} y = f(x).$$

Tuliskan penyelesaian pelengkap, penyelesaian khusus dan Wronskian.

[100 markah]

Question 3

(a) According to Newton’s law of cooling/warming, the rate at which the temperature of a body changes is proportional to the difference between the temperature of a body and the temperature of the surrounding medium or the ambient temperature. Formulate the Newton’s law of cooling/warming.

(b) Using the formula obtained in (a), suppose a metal spoon whose initial temperature was $20°C$, is dropped into a large container of boiling water. It is known that the temperature increases $2°C$ in 1 second;

(i) How long will it take the spoon to reach $90°C$?

(ii) How long will it take the spoon to reach $98°C$?

Give your comment(s) on the results obtained.

[100 marks]

Soalan 3

(a) Berdasarkan hukum penyejukan/pemanasan Newton, kadar perubahan suhu badan adalah berkadaran dengan perbezaan antara suhu badan dan suhu medium sekeliling atau suhu udara sekitar. Rumuskan hukum penyejukan/pemanasan Newton.

(b) Menggunakan formula yang diperoleh di (a), andaikan sebatang sudu besi yang bersuhu awal $20°C$ *dijatuhkan ke dalam sebuah bekas besar yang mengandungi air mendidih. Diketahui bahawa suhu meningkat* $2°C$ *dalam 1 saat;*

(i) Berapa lamakah masa yang diambil untuk sudu tersebut mencapai $90°C$?

(ii) Berapa lamakah masa yang diambil untuk sudu tersebut mencapai $98°C$?

Berikan komen(-komen) bagi keputusan yang diperoleh.

[100 markah]

Question 4

(a) Explain the differences between the analytical solution and the numerical solution in solving differential equation problem.

(b) Consider the first order initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

where $n = 0,1,2, \ldots$, h is the time step size and points along the x-axis are given by $x_n = x_0 + nh$. Write the formula for the fourth order Runge-Kutta method (RK4).

(c) Using the formula obtained in (b), consider an example of a differential equation

$$\frac{dy}{dx} = y - x^2 + 1, \quad y(0) = 0.5.$$

(i) Solve this problem by using $h = 0.5$ and $0 \leq x \leq 2$.

(ii) The exact solution for this problem is $y = x^2 + 2x + 1 - \frac{1}{2}e^x$. Comment on the errors obtained.

[100 marks]

Soalan 4

(a) Terangkan perbezaan antara penyelesaian analitik dan penyelesaian berangka dalam menyelesaikan masalah persamaan pembezaan.

(b) Pertimbangkan masalah nilai awal peringkat pertama

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

di mana $n = 0,1,2, \ldots$, h *adalah saiz langkah dan titik-titik di sepanjang paksi-x diberikan sebagai* $x_n = x_0 + nh$. *Tuliskan rumus bagi kaedah Runge-Kutta peringkat keempat (RK4).*

(c) Menggunakan formula yang diperoleh di (b), pertimbangkan contoh suatu persamaan pembezaan

$$\frac{dy}{dx} = y - x^2 + 1, \quad y(0) = 0.5.$$

(i) Selesaikan masalah tersebut dengan menggunakan $h = 0.5$ *and* $0 \leq x \leq 2$.

(ii) Penyelesaian tepat bagi masalah ini adalah $y = x^2 + 2x + 1 - \frac{1}{2}e^x$. *Komen tentang ralat yang diperoleh.*

[100 markah]

Question 5

(a) Write the general form of the power series solution of a differential equation

$$y'' + P(x)y' + Q(x)y = f(x).$$

(b) Use the power series method to solve the differential equation

$$(x^2 + 1)y'' + xy' = y.$$

(c) Solve the initial value problem

$$\frac{dy}{dt} + 3y = 13\sin(2t)\,, \quad y(0) = 6,$$

by using Laplace transform method. (Note: Refer the Laplace table given at the final page).

[100 marks]

Soalan 5

(a) Tuliskan bentuk am penyelesaian siri kuasa bagi persamaan pembezaan

$$y'' + P(x)y' + Q(x)y = f(x).$$

(b) Gunakan kaedah siri kuasa untuk menyelesaikan persamaan pembezaan

$$(x^2 + 1)y'' + xy' = y.$$

(c) Selesaikan masalah nilai awal

$$\frac{dy}{dt} + 3y = 13\sin(2t)\,, \quad y(0) = 6,$$

menggunakan kaedah jelmaan Laplace. (Nota: Rujuk jadual Laplace yang diberikan di muka surat terakhir).

[100 markah]

Table of Laplace Transform.

$f(x), \quad x \geq 0$	$F(s)$
$\delta(x)$	1
$u(x)$	$\frac{1}{s}$
x	$\frac{1}{s^2}$
x^n	$\frac{n!}{s^{n+1}}$
e^{-ax}	$\frac{1}{s+a}$
$\sin bx$	$\frac{b}{s^2+b^2}$
$\cos bx$	$\frac{s}{s^2+b^2}$
$e^{-ax}\sin \omega x$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-ax}\cos \omega x$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{df(x)}{dx}$	$sF(s)-f(0)$
$\int_0^x f(x)dx$	$\frac{1}{s}F(s)$
$f(x-a)$	$e^{-as}F(s)$

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